Backstepping: A method to Design of Controllers and Observers for Partial Differential Equations

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The method known as backstepping for Partial Differential Equations (PDEs), as it is known today, was first introduced in the seminal work of Smyshlyaev and Krstic [1]. Their approach, first developed for a general 1-D linear reaction-diffusion-advection PDE, is based on a constructive strategy of first design (in the continuum setting) and then discretize (for implementation and simulation). The method has three main ingredients:

1. the selection of a target system which verifies the desired properties (typically stability, proven with a Lyapunov function), but still closely resembles the original system;
2. the use of an invertible integral transformation (the backstepping transformation), that maps the original plant into the target system in the appropriate functional spaces;
3. and the kernel equations, which are determined from the original and target systems and the transformation, and whose solution determine the kernel of the integral transformation. These equations typically are of Goursat type, namely hyperbolic boundary-problems on triangular domains, and can be usually proven solvable by transforming them to integral equations and then using the method of successive approximations.

These ingredients are closely connected. Judiciously choosing the target system will result in solvable kernel equations and an invertible transformation. On the other hand, an ill-chosen target system typically results in kernel equations that cannot be solved or even properly formulated, or a non-invertible transformation.

Recent results include stabilization of coupled hyperbolic systems [2], stabilization of coupled parabolic systems [3] and extensions to higher-dimensional domains [4]. These results will be discussed to show how the three ingredients (target system, transformation, kernel equations) change for different applications. Finally, some challenging open problems will be presented.

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References


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1 Not to be confused with finite-dimensional backstepping for Ordinary Differential Equations, which is mainly a nonlinear method.