Bifurcation theory and catastrophic shifts: from cancer evolution to complex ecosystems

Josep Sardanyés*

The presence of qualitative changes in nonlinear systems’ dynamics is a key research topic with deep implications. Such implications range from potential tumor extinction under targeted cancer therapies to catastrophic ecological shifts in the face of environmental change. Bifurcations can govern these catastrophic shifts, which can be of different nature. In this talk we will discuss different bifurcations responsible for abrupt (discontinuous) transitions towards extinctions. First, we will comment on recent results reported for an ODEs model describing the dynamics of competition between healthy cells and tumor cell populations with heterogeneous phenotypes. The model, inspired in so-called targeted cancer therapies, indicates that a catastrophic shift may be forced using combined therapies. A fine mathematical analysis reveals a novel type of bifurcation, that we name as a trans-heteroclinic bifurcation. Such a bifurcation involves the exchange of stability between two isolated, distant fixed points that have an heteroclinic connection [1].

Second, a mathematical model describing the dynamics of autocatalysis is presented as a one-dimensional differential equation to account for the fragility in metapopulations with facilitation. Two different types of dynamical systems are used to explore the dynamics and the transitions for this model, which aims to describe a fragmented population under processes of facilitation and extinction. On one hand, a mean field model based on Levins’ metapopulations will be analyzed. The model reads:

\[ f_\mu(x) = \frac{dx}{dt} = cx^2(1 - D - x) - e x. \]

The state variable, \( x(t) \), is the number of individuals of the population, and constants \( c > 0 \) and \( e > 0 \) are colonization and extinction rates. Parameter \( 0 \leq D \leq 1 \) introduces the fraction of habitat destroyed. The dynamics of Eq. (1) indicates a bistable scenario and a catastrophic shift governed by a saddle-node bifurcation of fixed points. Further investigations by building a potential function given by

\[ V_\mu(x) = - \int f_\mu(x) dx = -cx^2 \left( -\frac{r}{2} + \frac{1}{3} x - \frac{1}{4} x^2 \right), \]

indicate the nature of this bistable scenario.

On the other hand, an agent-based model considering spatial correlations of the system described above is also explored. By using stochastic cellular automata (CA) models we determine how this catastrophic extinction depends on the model parameters as well as in the colonization strategy.

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References


*Centre de Recerca Matemàtica, Campus de Bellaterra, Edifici C 08193 Bellaterra, Barcelona, Spain. Email: jsardanyes@crm.cat