PDE models for total value adjustment in European and American options

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Since the beginning of the last crisis, financial entities have made an important effort on managing the different aspects of risk. So, different adjustments (XVA) on risk–free derivative value are now included in derivative contracts. In particular, the credit value adjustment (CVA) refers to the increment on the price of a contract due to the possibility of default of one (or both) of the parts. Adjustments on debit (DVA) and funding (FVA) are also important issues included in XVA.

Among the different approaches to XVA computing, we follow the one based on partial differential equations (PDE). We study a general framework in which stochastic default intensities are assumed for both counterparties (investor and issuer) [4]. In the present work, a zero risk intensity from the issuer is assumed and as a consequence a PDE model depending on two stochastic factors is obtained:

\[
\begin{aligned}
\frac{\partial U}{\partial t} + \frac{1}{2}(\sigma_S^2 S^2 \frac{\partial^2 U}{\partial S^2}) + \frac{1}{2}(\sigma_h^2 \frac{\partial^2 U}{\partial h^2}) + \\
+ \rho \sigma_S \sigma_h S \frac{\partial U}{\partial h} &+ (r - q) S \frac{\partial U}{\partial S} + (\mu^h - M \sigma_h^2) \frac{\partial U}{\partial h},
\end{aligned}
\]

where:

\[
\begin{aligned}
\frac{\partial U}{\partial t} - \mathcal{L}_S U - f_U &= (V + U)+ h \\
U(T, S, h) &= 0
\end{aligned}
\]

Moreover, we propose a set of numerical methods for solving the PDEs. In European-style options, semi-lagrangian methods for time discretization, finite elements for spatial discretization and fixed point methods to cope with the global nonlinear feature are proposed. In the case of American–style options, which leads to obstacle problems, an Augmented Lagrangian Active Set method is additionally implemented. Finally, some 1–d and 2–d numerical results will be presented.

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References


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