A note on a family of singly implicit peer methods for solving stiff IVPs

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We consider fixed step size $s$-stage singly implicit two-step peer methods for solving stiff Initial Value Problems (IVPs)

\begin{align}
\begin{cases}
y'(t) = f(t, y(t)), & t \in [t_0, t_0 + T] \\
y(t_0) = y_0 \in \mathbb{R}^d
\end{cases}
\end{align}

in which the step $t_n \to t_{n+1} = t_n + h$ is given by the formulae

\begin{align}
Y_{n+1,i} = \sum_{j=1}^{s} b_{i,j} Y_{n,j} + h \sum_{j=1}^{i-1} g_{i,j} f_{n+1,j} + h \gamma f(t_{n+1,i}, Y_{n+1,i}),
\end{align}

where $h$ is the step size, $Y_{n,j}$ are approximations to the solution $y(t)$ of (1) at the points $t_{n,j} = t_n + c_j h$, $f_{n+1,j} = f(t_{n+1,j}, Y_{n+1,j})$. Further, $\{c_i\}_{i=1}^{s}$ is a set of non confluent nodes ($c_i \neq c_j$ for all $i \neq j$), usually $0 < c_1 < c_2 \ldots < c_s = 1$ and $B = (b_{ij})$, $G_0 = (g_{ij})$ and $\gamma$ are the constants that define the method.

Methods of this type have been proposed by Weiner, Schmitt, Podhaisky and coworkers in [1], [2] as an alternative to the classical families of linear multistep and Runge-Kutta (RK) methods with the purpose to improve the accuracy and stability properties of standard methods in both families. Also different families of parallel two-step peer methods have been considered in [4],[5] for the numerical solution of (1). The aim of this note is to present a family of $s$-stage ($s \geq 2$) methods of type (2) with (stage) order $(s-1)$, optimally zero stable. We will see that these two conditions determine the available parameters of $G_0$ and $B$. Then, we study their absolute stability as function of the free parameter $\gamma$ proving that for all $s \leq 8$ there are intervals $(\gamma_{-}^{s}, \gamma_{+}^{s})$ so that for all $\gamma$ in this interval the corresponding methods are $L$-stable. In addition, it will be shown that such a methods can be viewed as a composition of BDF type multistep methods and then the well developed implementation of BDF methods for stiff problems [3] can be applied to this family of methods.

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References


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